Viscous-Gravity Spreading of Oil on Water

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Spreading of oil on calm water is analyzed for the second stage of spread, which is controlled by a balance between gravity and viscous forces. The analysis, using the integral boundary-layer method, agreed well with a numerical solution for the case of unidirectional spread. The analysis is extended to include axisymmetric spread for which there is no sound theoretical treatment presented in the literature. The approximations made in the model are discussed based on the lubrication approximation and order-of-magnitude arguments.

Introduction

Oil spilled on sea water is subject to weathering due to several factors (Cormack, 1986; Mackay and McAuliffe, 1988; Spaulding, 1988; Fingas et al., 1996), including spreading which causes a significant increase in the area of the oil spilled. Models used for describing the dynamics of spreading are reviewed by Hoult (1972), Palczynski (1987), and Spaulding (1988). In particular, Fay's treatment (1969, 1971) has found wide acceptance (Spaulding, 1988), and has been tested experimentally. Fay's model basically shows the existence of three spreading regimes. Initially, spreading is promoted by gravity and counteracted by the action of inertia forces. This first stage is followed by a stage in which viscous forces replace inertia as the predominant resisting force. Finally, there is a stage where surface-tension becomes the predominant driving force.

In our study, we consider the second stage of spreading in which gravity forces are balanced by viscous forces. Although Fay's theoretical treatment gives the power-law form of the spill-size time dependency, it does not provide the spreading laws prefactors, which were determined experimentally for the only case of unidirectional spread (Hoult, 1972). In further theoretical development, Hoult and Suchon (Hoult, 1972) proposed a similarity solution in which the boundary layer is assumed to depend on time exclusively. Their model still failed to determine the unknown prefactors. Buckmaster (1973) noted that Hoult and Suchon's model is not acceptable in the vicinity of the spill edge, where the boundary-layer thickness is expected to be zero. Solving numerically by integration from the leading edge, Buckmaster found the unidirectional-spread prefactor to deviate by 17.3% from the experimental value reported by Hoult (1972).

In our treatment, the lubrication-theory approximation and the von Karman-Pohlhausen integral technique are used to model and solve the flow problem in the oil and water phases. The similarity solution of Hoult and Suchon is modified to allow for the boundary layer to be a function of both time and position. The boundary-layer equations are solved by applying the integral boundary-layer method, which, in conjunction with the solution of the oil governing equations, yields the spreading-law prefactors for two geometrical configurations: unidirectional and axisymmetric spreads, which are both needed in fate modeling (Shen et al., 1987).

Results are compared with the experimental work presented by Hoult (1972) and the theoretical work of Buckmaster (1973), both available for the only case of unidirectional spread. Finally, the approximations used in the present model are discussed on the basis of the present findings in comparison with the experimental data reported by Hoult (1972).

Governing Equations

In the second stage of spreading, viscous forces come into play; therefore, a complete formulation of the equations governing the flow is needed for both the oil and water phases. A spreading spill, assumed symmetric, is shown in Figure 1. If spreading is unidirectional, then x represents the horizontal Cartesian coordinate in the direction of spread, and s is the vertical distance from the mean water level. On the other hand, if axisymmetric spread is considered, then spread occurs radially, and x and s represent cylindrical coordinates. In the second stage of spreading, inertia forces become negli-

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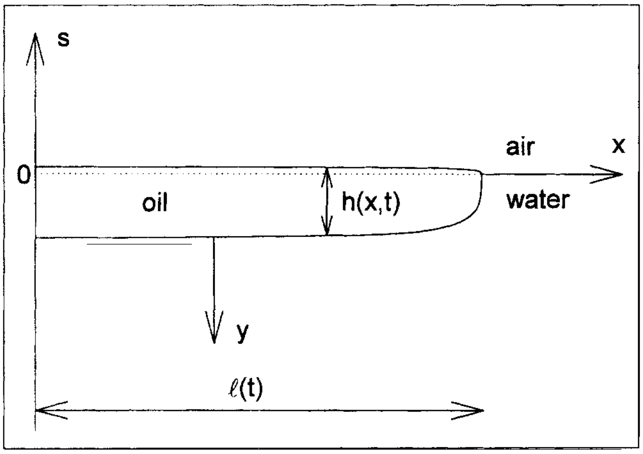

Figure 1. Shape of the righthand side of a symmetric oil spill spreading on water.

gible compared to gravity effects. In addition, the oil layer being thin and presenting small interface slopes, we apply the lubrication theory. With these approximations, variations of pressure p and oil velocity U are given by

$$0 = \mu_o \frac{\partial^2 U}{\partial s^2} - \frac{\partial p}{\partial x}, \tag{1}$$

and

$$0 = -\frac{\partial p}{\partial s} - \rho_o g, \tag{2}$$

in which g denotes the gravitational acceleration, and ρ_o and μ_o represent the oil density and viscosity, respectively.

The edge of the spill is defined as the line along which the thickness h is zero. Making a differential material balance yields variations of h with time t

$$\frac{\partial h}{\partial t} = -\frac{1}{x^\zeta}\frac{\partial}{\partial x}\left(x^\zeta \int_0^h U\, ds\right), \tag{3}$$

in which ζ is zero in the case of unidirectional spread, and equal to one in the case of axisymmetric geometry.

The oil-thickness profile is constrained to satisfy

$$V = \int_0^\ell (2\pi x)^\zeta h\, dx, \tag{4}$$

where ℓ is the spill size as defined in Figure 1, and, as in Hoult's notations, V denotes half of the volume per unit length in the case of unidirectional spread, and the total spill volume in the axisymmetric case.

When viscous effects are important, solving the oil equations requires solving the water continuity and momentum equations, due to the fact that there is coupling between the oil- and water-governing equations. The coupling results from the continuity of the velocity profile (no-slip condition)

$$u = U \qquad \text{at } y = 0, \tag{5}$$

and the tangential stress condition

$$\mu_w \frac{\partial u}{\partial y} = \mu_o \frac{\partial U}{\partial y} \qquad \text{at } y = 0, \tag{6}$$

in which u represents the horizontal component of the water velocity in the direction of spread, μ_w denotes the water viscosity, and y is the vertical distance from the oil-water interface, as shown in Figure 1.

The movement of the oil spill, driven by gravity, induces motion in the water phase, which in return exerts drag on oil. The movement is confined to the boundary layer (Fay, 1969, 1971; Hoult, 1972), and the water momentum equations are approximated by the boundary-layer form (Hoult, 1972)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu_w \frac{\partial^2 u}{\partial y^2}, \tag{7}$$

and

$$0 = -\frac{\partial p}{\partial s} - \rho_w g, \tag{8}$$

where ρ_w is the density of water, v is the vertical component of the water velocity perpendicular to the direction of spread, and ν_w is the kinematic viscosity of water. Variations of u and v are related by the continuity equation

$$\frac{1}{x^\zeta}\frac{\partial (x^\zeta u)}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{9}$$

The value of the fraction Δ of the oil thickness which floats above the mean water surface was derived by Hoult by making simple hydrostatic calculations

$$\Delta = \frac{\rho_w - \rho_o}{\rho_w}. \tag{10}$$

The pressure p in the oil phase is obtained by integrating Eq. 2 from the air-oil interface where the pressure p_a is atmospheric

$$p = p_a + \rho_o g[\Delta h - s]. \tag{11}$$

Substituting into Eq. 1 and integrating, while using the tangential stress condition

$$\frac{\partial U}{\partial s} = 0 \qquad \text{at } s = \Delta h, \tag{12}$$

gives

$$\mu_o \frac{\partial U}{\partial s} - \rho_w \Delta(1-\Delta) g (s - \Delta h) \frac{\partial h}{\partial x} = 0. \quad (13)$$

Applying Eq. 13 at the oil-water interface, while using Eq. 6 and neglecting Δ compared to 1, yields

$$\mu_w \frac{\partial u}{\partial s}\bigg|_{y=0} + \rho_w \Delta g h \frac{\partial h}{\partial x} = 0, \quad (14)$$

which is consistent with the result of Hoult and Suchon (Hoult, 1972).

Dimensionless Form

Viscous-gravity spreading is preceded by the inertia-gravity stage. Time t and spill size ℓ are normalized using estimated values for transition time T and transition spill size L, which are calculated by equating the spreading laws for the first and second stages, as given by Fay's (1969, 1972) order-of-magnitude analysis.

The order of magnitudes for the two predominant forces in the second stage of spreading are presented by Fay (1969, 1971) and Hoult (1972) as

$$\text{gravity force} \sim (\rho_w \Delta g h_c) h_c \ell^{\zeta}, \quad (15)$$

and

$$\text{viscous force} \sim \left[\mu_w (\ell t^{-1})(\nu_w t)^{-1/2} \right] \ell^{\zeta+1}, \quad (16)$$

in which h_c is a characteristic thickness satisfying the conservation-of-volume condition

$$V \sim h_c \ell^{\zeta+1}. \quad (17)$$

Equating Eqs. 15 and 16, while eliminating h_c by making use of Eq. 17, yields the viscous-gravity spreading law of Fay (1969, 1971)

$$\ell \sim \left(\frac{\Delta g V^2}{\nu_w^{1/2}} t^{3/2} \right)^{1/(2\zeta+4)}, \quad (18)$$

in which the missing proportionality constant needs to be determined. In the first stage of spreading, the inertia force which is of order (Fay, 1969, 1971)

$$\text{inertia force} \sim \rho_w (\ell t^{-2}) h_c \ell^{\zeta+1}, \quad (19)$$

is predominant compared to viscous forces.

In the transition from the inertia-gravity stage to the viscous-gravity one, the inertia and gravity forces become of the same order of magnitude. Therefore, equating Eqs. 16 and 19, while making use of Eqs. 17 and 18, yields

$$T = \left[(\Delta g)^{-(2\zeta+2)} V^4 \nu_w^{-(\zeta+3)} \right]^{1/(5\zeta+7)}, \quad (20)$$

and

$$L = \left[\Delta g V^5 \nu_w^{-2} \right]^{1/(5\zeta+7)}. \quad (21)$$

L and T being determined, we define dimensionless variables as follows

$$\bar{t} = t/T; \quad \bar{x} = x/L; \quad \bar{\ell} = \ell/L; \quad \bar{y} = y/\sqrt{\nu_w T};$$
$$\bar{h} = h/(V/L^{\zeta+1}); \quad \bar{u} = u/(L/T); \quad \bar{v} = v/\sqrt{\nu_w/T};$$
$$\bar{U} = U/(L/T). \quad (22)$$

In terms of the new variables, the oil momentum equation (Eq. 14) takes the form

$$-\frac{\partial \bar{u}}{\partial \bar{y}}\bigg|_{\bar{y}=0} + \bar{h} \frac{\partial \bar{h}}{\partial \bar{x}} = 0, \quad (23)$$

and, neglecting the oil velocity gradient in the vertical direction, the oil continuity Eq. 3 becomes

$$\frac{\partial \bar{h}}{\partial \bar{t}} = -\frac{1}{\bar{x}^{\zeta}} \frac{\partial (\bar{x}^{\zeta} \bar{U} \bar{h})}{\partial \bar{x}}. \quad (24)$$

In the same way, the conservation of the total oil volume and the water momentum and continuity equations are also expressed in terms of the new variables to give

$$\int_0^{\bar{\ell}} (2\pi \bar{x})^{\zeta} \bar{h}\, d\bar{x} = 1, \quad (25)$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}, \quad (26)$$

and

$$\frac{1}{\bar{x}^{\zeta}} \frac{\partial (\bar{x}^{\zeta} \bar{u})}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0. \quad (27)$$

Solution

Hoult and Suchon (Hoult, 1972) proposed a similarity solution in which combined variables are determined based on order-of-magnitude analysis. In the present treatment, we use the same following combined variables

$$\eta = \bar{x} \bar{t}^{-(3-\zeta)/8}; \quad \bar{U} = \bar{x} \bar{t}^{-1} C; \quad \bar{h} = H(\eta) \bar{x}^{-(1+\zeta)}. \quad (28)$$

However, to approximate the flow in the water phase, we use the following polynomial of the sixth degree

$$\bar{u}/\bar{U} = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4 + a_5 z^5 + a_6 z^6, \quad (29)$$

which is function of

$$z = \bar{y}/\delta, \quad (30)$$

in which the boundary-layer thickness δ is allowed to be function of both time and position, instead of time only

$$\delta = \tilde{\delta}(\eta)\sqrt{\bar{t}}. \quad (31)$$

Substituting Eq. 28 into the oil continuity Eq. 24 yields the value of C

$$C = (3 - \zeta)/8, \quad (32)$$

which is consistent with the expression reported by Hoult (1972).

In order to obtain the polynomial coefficients a_i ($i = 0, 1, \ldots, 6$), we apply the following boundary conditions evaluated at the oil-water interface and at the edge of the boundary layer, respectively

$$\bar{u} = \bar{U}; \quad \frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2};$$

$$\frac{\partial}{\partial \bar{y}}\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = \frac{\partial^3 \bar{u}}{\partial \bar{y}^3} \quad \text{at } \bar{y} = 0, \quad (33)$$

and

$$\bar{u} = \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} = \frac{\partial^3 \bar{u}}{\partial \bar{y}^3} = 0 \quad \text{at } \bar{y} = \delta. \quad (34)$$

Integrating the water continuity Eq. 27, while making use of the fact that water does not penetrate in the oil phase

$$\bar{v} = 0 \quad \text{at } \bar{y} = 0, \quad (35)$$

gives

$$\bar{v} = -\frac{1}{\bar{x}^{\zeta}}\int_0^{\bar{y}} \frac{\partial}{\partial \bar{x}}(\bar{x}^{\zeta}\bar{u})d\bar{y}. \quad (36)$$

The boundary-layer thickness can be calculated by integrating the water momentum Eq. 26 between 0 and δ. Integrating by parts to eliminate $\bar{v}$, while using Leibnitz's formula along with Eqs. 34 and 35, we obtain

$$\frac{\partial}{\partial \bar{t}}\left(\int_0^{\delta} \bar{u}d\bar{y}\right) + \frac{\partial}{\partial \bar{x}}\left(\int_0^{\delta} \bar{u}^2 d\bar{y}\right) + \frac{\zeta}{\bar{x}}\int_0^{\delta} \bar{u}^2 d\bar{y} = -\left.\frac{\partial \bar{u}}{\partial \bar{y}}\right|_{\bar{y}=0}. \quad (37)$$

Substituting for $\bar{u}$ from Eq. 29, and making use of Eqs. 28, 30, and 31 yields

$$C\eta\tilde{\delta}\frac{d}{d\eta}\left[(\beta - \alpha)\tilde{\delta}\right] + \left[(2+\zeta)C\beta - \frac{\alpha}{2}\right]\tilde{\delta}^2 = -a_1, \quad (38)$$

in which

$$\alpha = \sum_0^6 \frac{a_i}{i+1}, \quad (39)$$

and

$$\beta = \sum_0^{12} \frac{b_i}{i+1}. \quad (40)$$

The b-terms in the foregoing equation are defined as the coefficients of the polynomial

$$\sum_0^{12} b_i z^i = \left(\sum_0^6 a_i z^i\right)^2. \quad (41)$$

The solution of Eq. 38 is subject to the condition

$$\tilde{\delta} = 0 \quad \text{at } \eta = \eta_m, \quad (42)$$

which expresses that the boundary-layer thickness is zero at the leading edge of the spreading oil spill, where η is maximum and equal to η_m.

On the other hand, the oil momentum equation yields

$$H\left[\eta\frac{dH}{d\eta} - (\zeta+1)H\right] = \frac{Ca_1\eta^{2\zeta+4}}{\tilde{\delta}}. \quad (43)$$

Letting

$$G = H^2\eta^{-2(\zeta+1)}, \quad (44)$$

substituting into Eq. 43, and integrating while using

$$H = 0 \quad \text{at } \eta = \eta_m, \quad (45)$$

gives

$$G = 2C\int_{\eta_m}^{\eta}\left(a_1\eta/\tilde{\delta}\right)d\eta. \quad (46)$$

Applying Eq. 28 along with Eqs. 20 to Eq. 22 shows that the spreading-law prefactor is equal to η_m. In terms of the normalized variable

$$\bar{\eta} = \eta/\eta_m, \quad (47)$$

Table 1. Coefficients of the Sixth-Order Polynomial Velocity Profile

Geometry	a_0	a_1	a_2	a_3	a_4
Unidirectional	1	$20(-16+\tilde{\delta}^2)/D$	$-5\tilde{\delta}^2/16$	$15(16\tilde{\delta}^2-\tilde{\delta}^4)/(4D)$	$5(1{,}280-56\tilde{\delta}^2+9\tilde{\delta}^4)/(8D)$
Axisymmetric	1	$2(-40+3\tilde{\delta}^2)/D$	$-3\tilde{\delta}^2/8$	$(40\tilde{\delta}^2-3\tilde{\delta}^4)/(2D)$	$(800-60\tilde{\delta}^2+9\tilde{\delta}^4)/(4D)$
	a_5	a_6	D		
Unidirectional	$(-3{,}840+32\tilde{\delta}^2-15\tilde{\delta}^4)/(4D)$	$5(1{,}024+3\tilde{\delta}^4)/(16D)$	$160-3\tilde{\delta}^2$		
Axisymmetric	$3(-160+4\tilde{\delta}^2-\tilde{\delta}^4)/(2D)$	$(640-8\tilde{\delta}^2+3\tilde{\delta}^4)/(8D)$	$40-\tilde{\delta}^2$		

we write Eq. 46 in the following form

$$\overline{G}(\overline{\eta})=G/\eta_m^2=2C\int_1^{\overline{\eta}}\left(a_1\overline{\eta}/\tilde{\delta}\right)d\overline{\eta}. \tag{48}$$

From Eqs. 20 to 22, 28, 44 and 48, it is clear that

$$\sqrt{\overline{G}}=\frac{1}{\eta_m}\left(\frac{\Delta g}{\nu_w^{1/2}V^2}t^{3/2}\right)^{1/4}h \qquad \text{for } \zeta=0, \tag{49}$$

and

$$\sqrt{\overline{G}}=\frac{1}{\eta_m}\left(\frac{\Delta^2 g^2}{\nu_w V^2}t^{3}\right)^{1/6}h \qquad \text{for } \zeta=1, \tag{50}$$

meaning that $\sqrt{\overline{G}}$ represents a dimensionless thickness.

Once $\overline{G}$ is calculated, the value of η_m can be obtained by substituting for $\overline{h}$ into Eq. 25, by making use of Eqs. 28, 44, and 48

$$\eta_m=\left[(2\pi)^{\zeta}\int_0^1\sqrt{\overline{G}}\,\overline{\eta}^{\zeta}d\overline{\eta}\right]^{-1/(\zeta+2)}. \tag{51}$$

Results and Discussion

The solution procedure starts by solving Eqs. 33 and 34 to determine the sixth-order polynomial coefficients. Next, we substitute into Eq. 39 to get α. Solving for the b-coefficients in terms of the a-coefficients gives β upon substitution into Eq. 40. The results obtained are given in Tables 1 and 2. The expressions for α and β are substituted into Eq. 38. This differential equation, which shows an infinite slope at the leading edge, can be solved by recasting it in the following form

$$C\overline{\eta}\left[\frac{1}{2}(\beta-\alpha)\frac{d\tilde{\delta}_s}{d\overline{\eta}}+\tilde{\delta}_s\frac{d}{d\overline{\eta}}(\beta-\alpha)\right]+\left[(2+\zeta)C\beta-\frac{\alpha}{2}\right]\tilde{\delta}_s=-a_1, \tag{52}$$

in which

$$\tilde{\delta}_s=\tilde{\delta}^2. \tag{53}$$

Starting with

$$\tilde{\delta}_s=0 \qquad \text{at } \overline{\eta}=1, \tag{54}$$

and integrating Eq. 52 yields the numerical solution for $\tilde{\delta}$, which upon substitution into Eq. 48 gives $\overline{G}$. After making use of Eq. 51, we obtain the value of the spreading-law prefactor η_m. Substituting for η_m, we finally get the following spreading laws

$$\ell=1.77\left(\frac{\Delta g V^2}{\nu_w^{1/2}}t^{3/2}\right)^{1/4} \qquad \text{for } \zeta=0, \tag{55}$$

and

$$\ell=1.15\left(\frac{\Delta g V^2}{\nu_w^{1/2}}t^{3/2}\right)^{1/6} \qquad \text{for } \zeta=1. \tag{56}$$

The value 1.77 obtained for η_m in the case of unidirectional spread deviates by 0.6% from the value obtained by Buckmaster (1973). In addition, the present analysis provides the axisymmetric spreading-law prefactor.

The water velocity profiles are plotted in Figures 2a and 2b along with the boundary-layer thickness profiles. We can notice that the boundary-layer thickness is constant in the core of the oil spill, and, near the rim, it declines to reach zero at the leading edge. In the work of Hoult, the boundary layer is assumed to depend on time alone. As we have seen, this assumption is valid in the core only and ceases to be valid in the vicinity of the leading edge.

Dimensionless oil-thickness profiles are shown in Figure 3. A good agreement is obtained with the results of Buckmaster, which are available in the case of unidirectional spread

Table 2. Results for α and β

Geometry	α	β
Unidirectional	$(15{,}360-112\tilde{\delta}^2+3\tilde{\delta}^4)/(336D)$	$(1{,}487{,}667{,}200-29{,}818{,}880\tilde{\delta}^2+526{,}592\tilde{\delta}^4-1{,}248\tilde{\delta}^6+45\tilde{\delta}^8)/(329{,}472D^2)$
Axisymmetric	$(3{,}200-40\tilde{\delta}^2+\tilde{\delta}^4)/(280D)$	$(116{,}224{,}000-3{,}571{,}200\tilde{\delta}^2+72{,}960\tilde{\delta}^4-360\tilde{\delta}^6+9\tilde{\delta}^8)/(411{,}840D^2)$

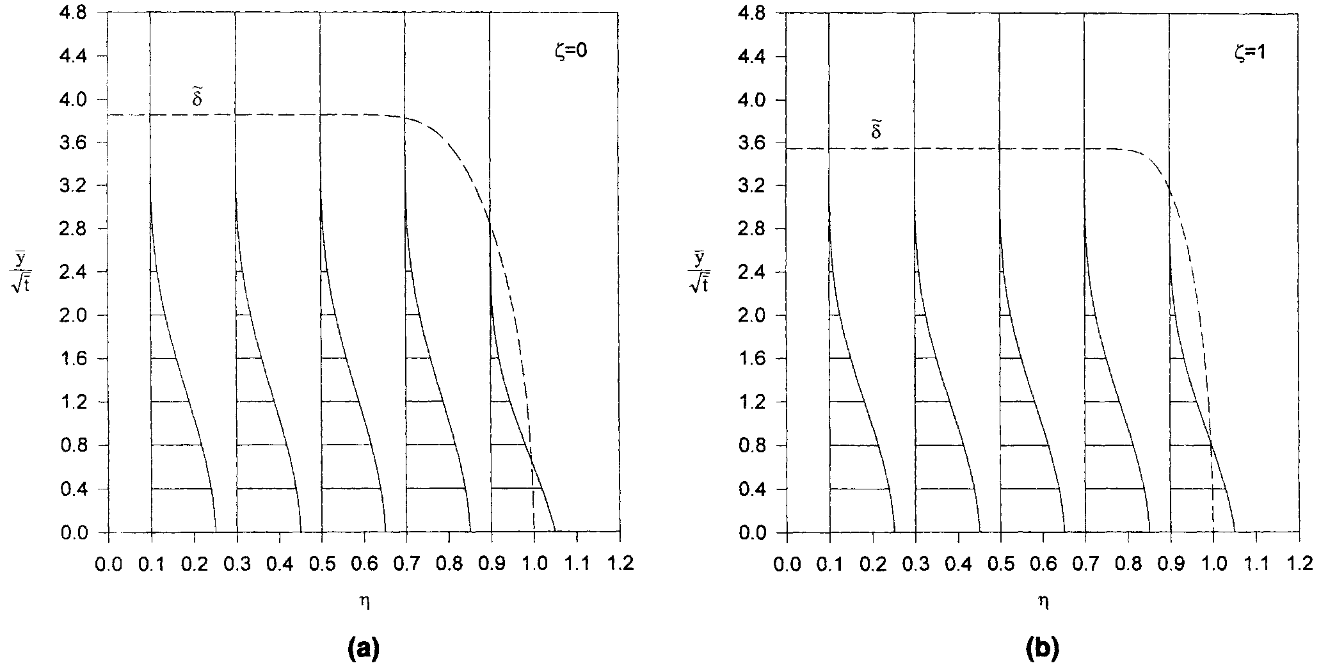

Figure 2. Water velocity profiles for (a) unidirectional and (b) axisymmetric spreads.

only. Our results for axisymmetric spread are presented in the same figure. For both spreading geometries, results show that the oil thickness does not show significant changes in the core of the oil spill; however, drastic changes are noticed in the near vicinity of the leading edge where the interface slope is infinite, meaning that the lubrication approximation is not strictly valid in the vicinity of the leading edge of the oil spill, which might eventually be causing, at least partly, the 18% deviation of η_m from the experimental value reported by Hoult (1972). However, the substantial agreement with the experimental value suggests that the lubrication approximation can be considered as acceptable. Another approximation used in this work neglects the oil velocity gradient in the vertical direction. To examine the validity of this approximation, we integrate Eq. 13 in order to obtain the difference between the oil velocities $U_{o\text{-}a}$ and $U_{o\text{-}w}$, evaluated at the oil-air and oil-water interfaces respectively

$$U_{o\text{-}a} - U_{o\text{-}w} = -\frac{\rho_w \Delta(1-\Delta)g}{2\mu_o} h^2 \frac{\partial h}{\partial x}. \tag{57}$$

Using order-of-magnitude analysis and neglecting Δ compared to 1, we estimate

$$\frac{U_{o\text{-}a} - U_{o\text{-}w}}{U_{o\text{-}w}} \sim \frac{\rho_w \Delta g h_c^3 t}{\mu_o \ell^2}. \tag{58}$$

Substituting for h_c and ℓ, by making use of Eqs. 18, 20, and 22, yields in terms of dimensionless time

$$\frac{U_{o\text{-}a} - U_{o\text{-}w}}{U_{o\text{-}w}} \sim \frac{\mu_w}{\mu_o} \bar{t}^{-(5\zeta+7)/(4\zeta+8)}. \tag{59}$$

Applying Eq. 59 at the estimated transition time shows that neglecting the oil velocity gradient in the vertical direction requires $\mu_o \gg \mu_w$, which is also a sufficient condition since the exponent of $\bar{t}$ in Eq. 59 is negative. This model limiting condition, which is derived based on the lubrication-theory approximation, agrees with the statement of Hoult and is in general not restrictive for crude oil spills since the viscosity ratio typically exceeds 20 (Hoult, 1972).

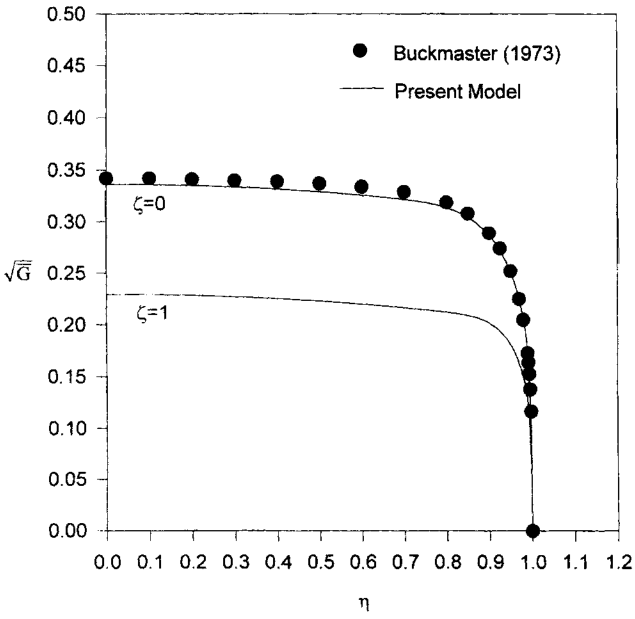

Figure 3. Oil-thickness profiles.

Notation

a_i = ith coefficient of the polynomial velocity profile
b_i = ith coefficient of the polynomial defined in Eq. 41
C = oil velocity profile constant, Eq. 28
D = denominator term defined in Table 1
g = gravitational acceleration
$G = \overline{G}\eta_m^2$
$\overline{G}$ = square of a dimensionless oil thickness, as defined in Eqs. 49 and 50
h = oil thickness
h_c = characteristic oil thickness
$\overline{h}$ = dimensionless oil thickness defined in Eq. 22
$H = \bar{x}^{1+\zeta}\overline{h}$
ℓ = spill size as defined in Figure 1
$\bar{\ell}$ = dimensionless spill size, Eq. 22
L = estimated distance to the transition point
p = pressure
p_a = atmospheric pressure
s = vertical distance from the mean water level
t = time
$\bar{t}$ = dimensionless time, Eq. 22
T = estimated time to the transition from the inertia-gravity stage to the viscous-gravity stage
u = horizontal component of the water velocity
$\bar{u}$ = dimensionless u, Eq. 22
U = oil velocity
$U_{o\text{-}a}$ = oil velocity at the oil-air interface
$U_{o\text{-}w}$ = oil velocity at the oil-water interface
$\overline{U}$ = dimensionless U, Eq. 22
v = vertical component of the water velocity
$\bar{v}$ = dimensionless v, Eq. 22
V = half of the oil volume in the case of unidirectional spread and total spill volume in the axisymmetric case
x = horizontal position as defined in Figure 1
$\bar{x}$ = dimensionless x, Eq. 22
y = vertical distance from the oil-water interface
$\bar{y}$ = dimensionless y, Eq. 22
z = normalized value of $\bar{y}$, Eq. 30

Greek letters

α = defined in terms of the polynomial coefficients a_i in Eq. 39
β = defined in terms of the polynomial coefficients b_i in Eq. 40
δ = boundary layer thickness divided by $\sqrt{\nu_w T}$
$\bar{\delta} = \delta/\sqrt{\bar{t}}$
$\bar{\delta}_s$ = square of $\bar{\delta}$
Δ = fraction of the oil thickness floating above the mean water surface, Eq. 10
η = combined variable, Eq. 28
η_m = maximum and leading-edge value of η
$\bar{\eta}$ = normalized value of η, Eq. 47
μ_o = oil viscosity
μ_w = water viscosity
ν_w = kinematic viscosity of water
ρ_o = oil density
ρ_w = water density
ζ = 0 for the case of unidirectional spread, and = 1 for the axisymmetric case

Literature Cited

Buckmaster, J., "Viscous-Gravity Spreading of an Oil Slick," *J. Fluid Mech.*, **59**, 481 (1973).

Cormack, D., *Response to Oil and Chemical Marine Pollution*, Elsevier Applied Science Publishers, London, p. 23 (1986).

Fay, J. A., "The Spread of Oil Slicks on a Calm Sea," *Oil on the Sea*, D. P. Hoult, ed., Plenum, New York, p. 53 (1969).

Fay, J. A., "Physical Processes in the Spread of Oil on a Water Surface," *Proc. of Joint Conf. on Prevention and Control of Oil Spills*, American Petroleum Institute, Washington, DC, 463 (1971).

Fingas, M., P. Jokuty, and B. Fieldhouse, "Oil Spill Behaviour and Modeling," *Proc. of Eco-Informa '96. Global Networks for Environmental Information*, **1**, Lake Buena Vista, FL, 471 (1996).

Hoult, D. P., "Oil Spreading on the Sea," *Ann. Rev. Fluid Mech.*, **4**, 341 (1972).

Mackay, D., and C. D. McAuliffe, "Fate of Hydrocarbons Discharged at Sea," *Oil Chem. Pollut.*, **5**, 1 (1988).

Palczynski, R. J., "Model Studies of the Effect of Temperature on Spreading Rate of a Crude Oil on Water," *Oil in Freshwater*, J. H. Vandermeulen and S. E. Hrudey, eds., Pergamon Press, Oxford, p. 22 (1987).

Shen, H. T., P. D. Yapa, and M. E. Petroski, "A Simulation Model for Oil Slick Transport in Lakes," *Water Resources Res.*, **23**, 1949 (1987).

Spaulding, M. L., "A State-of-the-Art Review of Oil Spill Trajectory and Fate Modeling," *Oil Chem. Pollut.*, **4**, 39 (1988).

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